

TEMPLATE METHODS FOR EFFICIENT MICROWAVE FILTER DESIGN

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Abstract

A method is presented to design a commensurate microwave filter, containing stubs, unit elements and arbitrary finite attenuation poles in such a way that it satisfies arbitrary system attenuation requirements in an optimal way.

1. Introduction

A filter may be called efficient, if it satisfies the system requirements with a minimal complexity. System specifications normally are reflected in a maximum pass-band ripple or VSWR, and in a minimal attenuation pattern in the stopband(s). In principle, this pattern may have any shape, and it may also contain a number of points where the attenuation should be essentially infinite [fig 1]. In lumped filter theory, methods exist to obtain the filter elements in such a way that it satisfies all constraints in an optimal way¹. These so called template methods heavily rely on exact network synthesis techniques. They can be translated into the domain of microwave filters, however only after assessing the differences in design philosophy and in the elements available to build-up the filter chain. As will be shown, significant improvement will result in the filter performances in a number of design cases.

2. General design principles

Lumped filters derive their selectivity mainly from the inclusion of finite imaginary axis transmission zeros, (T.Z.) realised by resonating LC sections. In distributed commensurate filters, it is also desirable to include T.Z.'s that do not coincide with the quarter wave resonance frequency of the lines, and this in view of their superior steepness performance. However, one should realise that their inclusion may greatly complicate the structure; therefore, it is best to select from two alternatives: either use as few finite T.Z.'s as possible² or else use special physical configurations such as digital elliptic filters³ or stepped halfwave interdigital filters⁴. One should realise that in the second alternative, one essentially makes use of a lumped low-pass elliptic prototype, which has its finite transmission zero's determined in number and location from a constant minimum attenuation requirement over the whole stopband and from the relative width of a transition zone that separates passband and stopband(s). All commensurate distributed filters involve some irrational transform function, which maps the real frequency response into an equivalent Ω plane low or high pass behaviour. Examples of this transform are: (f_0 denotes

the central frequency of the filter)

$$\Omega = a \operatorname{tg} \frac{\pi}{2} \frac{f}{f_0} \quad (1)$$

$$\Omega = a \left[\operatorname{tg} \frac{\pi}{4} \frac{f}{f_0} - \frac{1}{\operatorname{tg} \frac{\pi}{4} \frac{f}{f_0}} \right] \quad (2)$$

(1) is the classical Richards transform, (2) is a modified version encountered with the stepped halfwave interdigital filter. Both functions have the common property, that, when applied to bandpass filters, both stopbands are mapped into the same Ω plane region. If the stopband attenuation below and above the passband differ, and there is no reason to forbid this, it is clear that the Ω plane specification to be adopted for further use should be the most stringent specification. In case of a bandstop filter, unsymmetric specification with respect to the central frequency lead to the same procedure. While (1) and (2), when applied to most classical filter configurations, lead to a high- or low-pass Ω plane prototype, it should be pointed out that this is no limitation to the presented theory: indeed, with little effort it is quite possible to extend the formula's to cope with band-pass behaviour in the Ω plane. While so far a strict translation of lumped filter theory can be used, it is quite clear that this is not possible for Ω plane prototypes having effective unit elements as part of their equivalent circuit. Indeed, since the u.e. has no counterpart in lumped filter theory, it will be essential to include it in the microwave template method.

3. Template functions including unit elements

Skipping all theoretical derivations and concentrating on the essentials of the procedure, we define the filter insertion loss function

$$A^{db}(\Omega^2) = -10 \log |S_{21}(\Omega)|^2 = 10 \log[1 + H(\Omega^2)] \quad (3)$$

$H(\Omega^2)$ can always be constructed to have the following properties:

- It is a nonnegative even function in Ω
- It has equal ripple behaviour in the pass-band
[ripple A_p db]
- It has m T.Z. at zero (H.P.) or infinity (L.P.) [LC - type T.Z.]
- It has q T.Z. at arbitrary finite Ω_k in the stopband
- The stopband behaviour is found from :

$$A_S^{db} = A_c + nT(\gamma - \gamma_p) + mT(\gamma) + \sum_{k=1}^q 2T(\gamma - \gamma_k)$$

$$\text{with : } A_c = 10 \log \frac{10^{A_p/4} - 1}{4} \quad (4)$$

$$T(\gamma) = 10 \log \coth \frac{|\gamma|}{2} \quad (6)$$

γ is a logarithmic transform of the stopband, which now runs along the positive γ axis.

$$\text{L.P. } \gamma = -\frac{1}{2} \ln(1 - \frac{\Omega_c^2}{\Omega^2}); \quad \gamma_k = -\frac{1}{2} \ln(1 - \frac{\Omega_c^2}{\Omega_k^2});$$

$$\gamma_p = -\frac{1}{2} \ln(1 + \Omega_c^2) \quad (7)$$

$$\text{H.P. } \gamma = -\frac{1}{2} \ln(1 - \frac{\Omega_c^2}{\Omega^2}); \quad \gamma_k = -\frac{1}{2} \ln(1 - \frac{\Omega_k^2}{\Omega_c^2});$$

$$\gamma_p = -\frac{1}{2} \ln(1 + \frac{1}{\Omega_c^2}) \quad (8)$$

$T(\gamma)$ is the so called template function [fig 2] : indeed $A_S(\gamma)$ is build up from adding a number of shifted, but except for a weighting constant, otherwise identical functions. Contributions to the attenuation are threefold :

- m templates centered at the γ axis origin, stemming from the LC transmission zero's [quarter wave resonances]. They only contribute with their right side.
- n templates centered at γ_p , stemming from the u.e. transmission zero's. Here also, only the right tail is contributing, and indeed only from the γ origin on. This shows clearly the situations where the u.e. contribution is important : since their effect is determined by $|\gamma_p|$, which should be as small as possible, for a maximum effectiveness, it is clear that in e.g. narrow bandpass filters [$\Omega_c \gg 1$], the u.e. are almost equivalent to LC sections. However, in typical wideband designs [$\Omega_c = 1$, e.g.] their contribution, though less pronounced, is still worthwhile.
- q double weighted templates, centered at γ_k . The γ_k points are transforms of the finite γ_k axis transmission zero's. Their contribution is really remarkable : indeed, both sides of the template contribute, with double weight if compared to an LC section. It should be pointed out that (3) keeps its passband properties irrespective of the choice of m, n, q and the actual locations Ω_k . Only the stopband attenuation is changed. Therefore, a procedure to select these parameters in an optimal way is entirely based on formulae (4) to (8), together with a strategy to accommodate the system specifications and the particular filter structure.

For instance, in those cases where no unit elements are included (e.g. digital elliptic filter prototypes), it is quite clear that relaxation of the specifications in that part of the stopband which falls out of the normal operation range of the equipment will lead to a less complex design, or else to steeper cut-off characteristics.

4. Final design

Let us consider fig.3, where a transformed $A_S(\gamma)$ specification is shown. If $A_S(\gamma)$ contains frequencies of infinite attenuation, this is reflected in a number of fixed templates of the Ω_k type. The other γ_k points, can be shifted freely over the axis; together with the numbers m and n , they provide the necessary degrees of freedom in order to satisfy $A_S(\gamma)$. Needless to say that the physical structure of the filter may impose constraints on the numbers m, n, q . The basic problem now is to select a minimal realisation, i.e. a realisation which minimizes $m+n+q$, subject to the eventual constraints among m, n and q . Intuitively this will call for a crowding of the γ_k in those regions where the attenuation requirements are high. If there are no specifications over a portion of the axis, one can use the γ_k to increase the attenuation near the passband, enhancing the filter steepness in this way. On a strict mathematical basis, approximation theory shows that an optimal solution will be found in the function

$$E = A(\gamma) - A_S(\gamma)$$

has all its minima equal and nonnegative. A suitable method to satisfy this criterium is based on the Remez algorithm which involves an iterative solution of a nonlinear system of equations¹. Once the solution has been obtained, the way is open to a direct synthesis of the filter. Needless to say that this involves having access to a general filter synthesis program. An existing distributed filter synthesis program⁵ has been modified in order to accommodate the template method. It incorporates all the essential steps necessary to synthesize the filter starting with the attenuation specifications.

5. Design example

Let us design a bandstop filter with 100% relative bandwidth. The ripple in the passband should be less than 2 db, while over 80% of the stopband, in a symmetrically around f_0 centered region, the attenuation should at least be 40 db [fig.4]. This leaves a skirt of about 10%, where the transition between passband and stopband can occur. The filter should be preferably symmetric, and the number of u.e. should be one less than the number of LC transmission zero's. As few finite axis transmission zero's should be used. A possible solution is shown on fig. 4. Three LC stubs and two u.e. are used. Only one T.Z. is used. The minimum attenuation is 42 db, while the 40 db width of the stopband is 84%. Other solutions are possible where one of the requirements is exactly met, the other then becoming better than specified. For instance, limiting the minimum attenuation to a strict 40 db, would increase the 40 db attenuation bandwidth to 86%. The characteristic function

$H(\Omega^2)$ for the cited case is :

$$H(\Omega^2) = \left[25.69 \frac{\Omega(\Omega^2 - .18533)(\Omega^2 - .62173)(\Omega^2 - .95492)^2}{(1 + \Omega^2)(\Omega^2 - 1.817)} \right]$$

The synthesized filter, with all impedances normalized to one ohm, is shown, with its element values on fig. 5. It is symmetric, and the element value's are such that it is directly realisable using series stubs. The resonating section can be realized by a stepped impedance double length stub.

6. Conclusion

The presented procedure is superior over existing procedures in matching the structure of a microwave filter to a prescribed attenuation specification. As such, it finds a minimal configuration, in which every element is designed to contribute in the most efficient way to build up the filter performance.

List of references

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